3.18.1. Semantic Problems: Validity

A. Translate each of the following arguments into formal language, and then use **truth tables** to decide whether the argument is **valid**.

1. We won't have both ice cream and cake. ∴ We'll have neither ice cream nor cake.

2. Jack isn't a bird who can fly. ∴ Either Jack isn't a bird, or Jack can't fly. (*Hint: see the remarks on relative clauses and negations in 3.10, Section 3*.)

3. Trixie won't hit the jackpot unless Elvis does. ∴ Elvis won't hit the jackpot unless Trixie does.

4. We won't have both a tax cut and increased spending on Logic research. We won't have increased spending on logic research unless we have a tax cut too. ∴ We won't have increased spending on Logic research, but we will have a tax cut.

5. Neko won't eat fish unless Suki does. Suki won't eat fish unless Neko does. Either Neko won't eat fish, or Suki won't. ∴ Neither Neko nor Suki will eat fish.

6. Rex won't pass the exam unless he does so without studying. ∴ It's not the case that Rex will both study and pass the exam.

7. Either Trixie and Elvis will both hit the jackpot, or neither of them will.
∴ ∴ Elvis won't hit the jackpot unless Trixie does.

8. We'll either have truffles or grog, but not both. Either we won't have truffles, or we'll have neither truffles nor grog. ∴ We'll have grog without having truffles.

9. Either Jake and Dr. Slim will both make it to Tijuana, or neither of them will. Either Jake or Dr. Slim will fail to make it to Tijuana. ∴ Either Jake will make it to Tijuana or Dr. Slim will, but not both.

B. Truth and Validity Puzzle. Suki, Neko, and Jack are on trial, and testify as follows.

> **Suki**: Neko is guilty, but Jack isn't.
> **Neko**: Suki isn't guilty unless Jack is.
> **Jack**: I'm not guilty, but either Suki or Neko is.

Use truth tables to answer the following questions.

1. If everyone told the truth, who is innocent and who is guilty?
2. If no one is guilty, who lied?
3. One person's testimony *follows validly* from someone else's – whose?

(Adapted from Kleene 1967/2002: 67, Exercise #14.2; attributed to Jerome Keisler)

C. Validity and Contradictions. Explain why a consistent set of premises can't entail a contradictory conclusion (that is: why a contradiction can only follow validly from an *inconsistent* set of premises).

D. **Validity and Tautologies**. Explain why a tautology only entails tautologies (that is: why the only kind of sentence that follows validly from a tautology is a tautology).

E. Validity and Inconsistent Sentences, Revisited. Explain why entailing a contradiction is the hallmark of an inconsistent set of sentences – something **all and only** the inconsistent sets do.

Note that Exercise C makes half of this case, by showing that entailing a contradiction is something **only** an inconsistent set of sentences can do.

> A contradiction follows validly from a set of premises **only if** that set of premises is inconsistent.

Strengthen that claim by explaining why the following is also true.

> **Every** inconsistent set of sentences entails a contradiction.

(*Hint: if the premises of an argument are inconsistent, why is a search for a validity counterexample doomed?*)

F. Explain why a set of sentences, S, is inconsistent if, and only if, both of the following arguments are valid.

{Set of Sentences S}	{Set of Sentences S}
∴ P	∴ ~P

(1) To show that **if** set S is inconsistent, both sentences follow validly, first show that **every sentence follows validly from an inconsistent set of premises**.

(*Hint: if the premises of an argument are inconsistent, why is a search for a validity counterexample doomed?*)

(2) To show that both “P” and “~P” follow validly **only if** the premises are inconsistent, prove that these two inferences won’t both be valid for any consistent set of premises.

(*Hint: note that “P” and “~P” are never true together in the same valuation. Use that fact, along with the definition of “consistent set,” to show that we we’re guaranteed a validity counterexample for one or the other of the two arguments when the premises form a consistent set.*)